## 1.1 Propositional Logic

1. Which of these are propositions? What are the truth values of those that are propositions?
   a) Do not pass go.
   b) What time is it?
   c) There are no black flies in Maine.

2. What is the negation of each of these propositions?
   a) Jennifer and Teja are friends.
   b) There are 13 items in a baker's dozen.
   c) Abby sent more than 100 text messages every day.
   d) 121 is a perfect square.

3. Suppose that Smartphone A has 256 MB RAM and 32 GB ROM, and the resolution of its camera is 8 MP; Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions.
   a) Smartphone B has the most RAM of these three smartphones.
   b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.
   c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.
   d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.
   e) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

4. Suppose that during the most recent fiscal year, the annual revenue of Acme Computer was 138 billion dollars and its net profit was 8 billion dollars, the annual revenue of Nadir Software was 87 billion dollars and its net profit was 5 billion dollars, and the annual revenue of Quixote Media was 111 billion dollars and its net profit was 13 billion dollars. Determine the truth value of each of these propositions for the most recent fiscal year.
   a) Quixote Media had the largest annual revenue.
   b) Nadir Software had the lowest net profit and Acme Computer had the largest annual revenue.
   c) Acme Computer had the largest net profit or Quixote Media had the largest net profit.
   d) If Quixote Media had the smallest net profit, then Acme Computer had the largest annual revenue.
   e) Nadir Software had the smallest net profit if and only if Acme Computer had the largest annual revenue.

5. Let $p$ and $q$ be the propositions

   $p$ : I bought a lottery ticket this week.
   $q$ : I won the million dollar jackpot.

   Express each of these propositions as an English sentence.
   a) $\neg p$ b) $p \vee q$ c) $p \to q$
   d) $p \wedge q$ e) $p \leftrightarrow q$ f) $\neg p \to \neg q$
   g) $\neg p \wedge \neg q$ h) $\neg p \vee (p \wedge q)$

6. Let $p$ and $q$ be the propositions "The election is decided" and "The votes have been counted," respectively. Express each of these compound propositions as an English sentence.
   a) $\neg p$ b) $p \vee q$
   c) $\neg p \wedge q$ d) $q \to p$
   e) $\neg q \to \neg p$ f) $\neg p \to \neg q$
   g) $p \leftrightarrow q$ h) $\neg q \vee (\neg p \wedge q)$

7. Let $p$, $q$, and $r$ be the propositions

   $p$ : You have the flu.
   $q$ : You miss the final examination.
   $r$ : You pass the course.

   Express each of these propositions as an English sentence.
   a) $p \to q$ b) $\neg q \leftrightarrow r$
   c) $q \to \neg r$ d) $p \vee q \vee r$
   e) $(p \to \neg r) \vee (q \to \neg r)$
   f) $(p \wedge q) \vee (\neg q \wedge r)$

8. Let $p$, $q$, and $r$ be the propositions

   $p$ : You get an A on the final exam.
   $q$ : You do every exercise in this book.
   $r$ : You get an A in this class.

   Write these propositions using $p$, $q$, and $r$ and logical connectives (including negations).
   a) You get an A in this class, but you do not do every exercise in this book.
   b) You get an A on the final, you do every exercise in this book, and you get an A in this class.
   c) To get an A in this class, it is necessary for you to get an A on the final.
   d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.
   e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
   f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

9. Determine whether these biconditionals are true or false.
   a) $2 + 2 = 4$ if and only if $1 + 1 = 2$.
   b) $1 + 1 = 2$ if and only if $2 + 3 = 4$.
   c) $1 + 1 = 3$ if and only if monkeys can fly.
   d) $0 > 1$ if and only if $2 > 1$.

10. Determine whether each of these conditional statements is true or false.
    a) If $1 + 1 = 2$, then $2 + 2 = 5$.
    b) If $1 + 1 = 3$, then $2 + 2 = 4$.
    c) If $1 + 1 = 3$, then $2 + 2 = 5$.
    d) If monkeys can fly, then $1 + 1 = 3$.

## 1.3 Propositional Equivalences

1. Use truth tables to verify the associative laws

   **a)** $(p \vee q) \vee r \equiv p \vee (q \vee r)$.

   **b)** $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$.

2. Use De Morgan's laws to find the negation of each of the following statements.

   **a)** Kwame will take a job in industry or go to graduate school.
   **b)** Yoshiko knows Java and calculus.
   **c)** James is young and strong.
   **d)** Rita will move to Oregon or Washington.

3. Show that each of these conditional statements is a tautology by using truth tables.

   **a)** $(p \wedge q) \rightarrow p$
   **b)** $p \rightarrow (p \vee q)$
   **c)** $\neg p \rightarrow (p \rightarrow q)$
   **d)** $(p \wedge q) \rightarrow (p \rightarrow q)$
   **e)** $\neg(p \rightarrow q) \rightarrow p$
   **f)** $\neg(p \rightarrow q) \rightarrow \neg q$

4. Show that each of these conditional statements is a tautology by using truth tables.

   **a)** $[\neg p \wedge (p \vee q)] \rightarrow q$
   **b)** $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
   **c)** $[p \wedge (p \rightarrow q)] \rightarrow q$
   **d)** $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

5. Show that each conditional statement in Exercise 9 is a tautology without using truth tables.

   (Exercise 3)

6. Show that each conditional statement in Exercise 10 is a tautology without using truth tables.

   (Exercise 4)

7. Use truth tables to verify the absorption laws.

   **a)** $p \vee (p \wedge q) \equiv p$
   **b)** $p \wedge (p \vee q) \equiv p$

8. Determine whether $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ is a tautology.

9. Determine whether $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology.

10. Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent.

**(To do this, either show that both sides are true, or that both sides are false, for exactly, the same combinations of truth values of the propositional variables in these expressions (whichever is easier).)**